Emittance Measurements in the XFEL

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Outline

- Formalism of emittance measurements
- Options for the lattice of the diagnostic sections in the XFEL
- Error Analysis
  - Statistical errors
  - Systematical errors
- Coupling measurements
- Summary,Conclusions and Outlook

The Formalism of emittance measurements

- From $\sigma(s) = M\sigma(s_0)M^T$ one obtains the relation

$$\sigma_{1,1}(s)^2 = \left(M_{1,1}^2; \;\; 2M_{1,1}M_{1,2}; \;\; M_{1,2}^2 \right) \begin{pmatrix} \sigma_{1,1}(s_0)^2 \\ \sigma_{1,2}(s_0)^2 \\ \sigma_{2,2}(s_0)^2 \end{pmatrix}$$

- Measurements of the beam sizes at three different locations allow to determine the initial beam matrix elements
- The projected emittance is given by

$$\epsilon_x = \sqrt{\sigma_{1,1}(s_0)^2 \cdot \sigma_{2,2}(s_0)^2 - \sigma_{1,2}(s_0)^2}$$

- More than three measurements allow least square fit

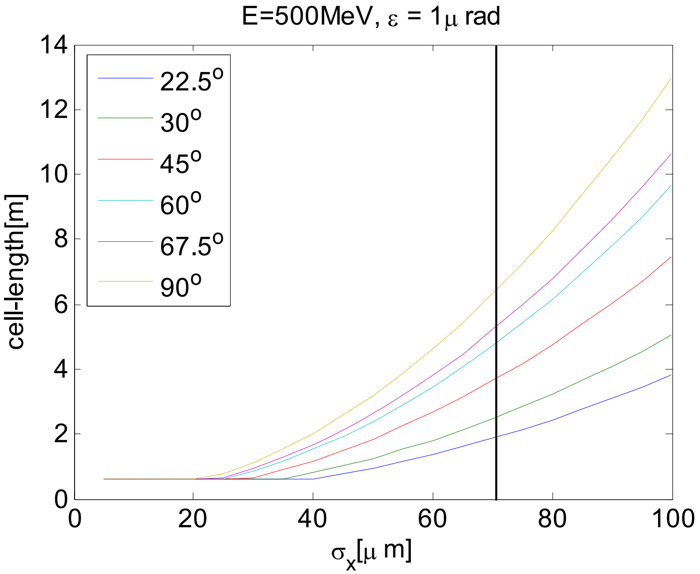
Lattice options

- Multi-monitor-method for online measurements
- Equal beam sizes at all stations reduce the resulting emittance error $\rightarrow$ FODO-lattices
- 180˚-periodicity of the design beta function guarantees 180˚-periodicity of the beam size for all initial conditions $\rightarrow$ Scan of 180˚ phase advance at regular intervals
- Phase advance options:

Ψ_{cell}	No. Meas.	No. cells	L^{*}_{tot}[m]	L^{**}_{tot}[m]
22.5°	8	7	13.1	26.7
30.0°	6	5	12.4	25.3
45.0°	4	3	11.0	22.4
60.0°	3	2	9.4	19.3
67.5°	8	5	26.0	53.2

$^{*}E = 500MeV$; $70\mu m$ beam size; $\epsilon = 1\mu rad$

$^{**}E = 2.0GeV$; $50\mu m$ beam size; $\epsilon = 1\mu rad$



Systematical errors

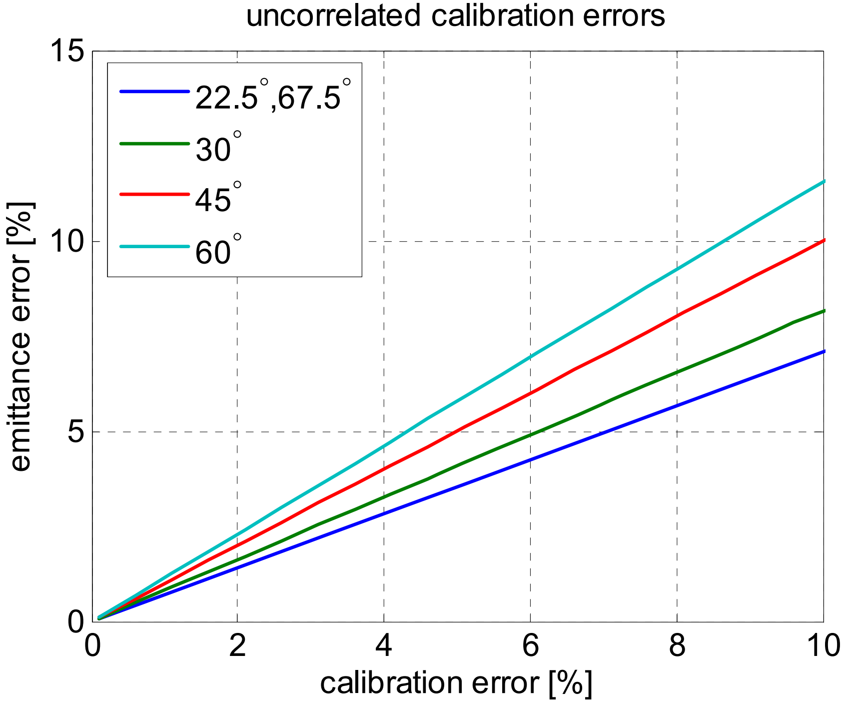
- Two types:
  - Measurement errors of the beam sizes
  - Deviations of the transfer matrices

- **Error sources:**
  - Calibration of the OTR-monitors
    - Statistically independent
    - Systematical:

$$\frac{\sigma_\epsilon}{\epsilon} = 2 \cdot \frac{\sigma_{x_{rms}}}{x_{rms}}$$

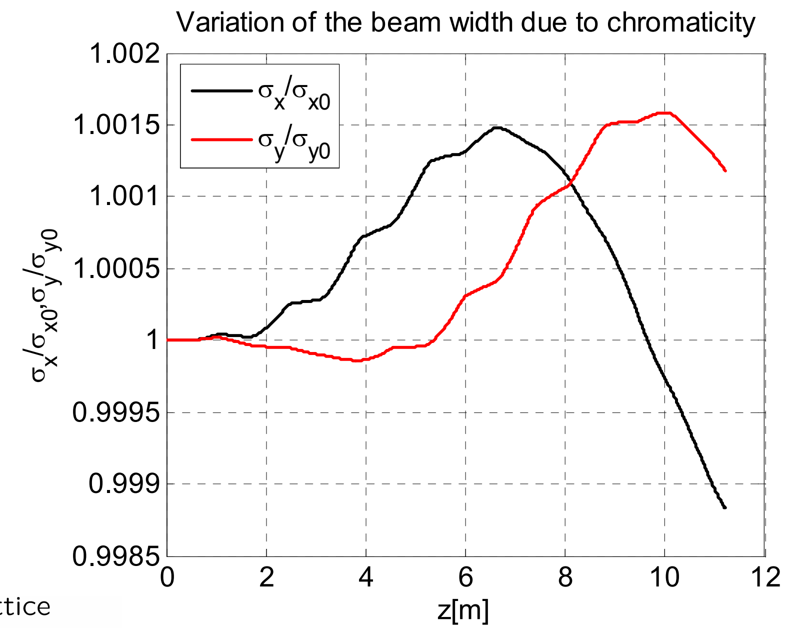
similar:

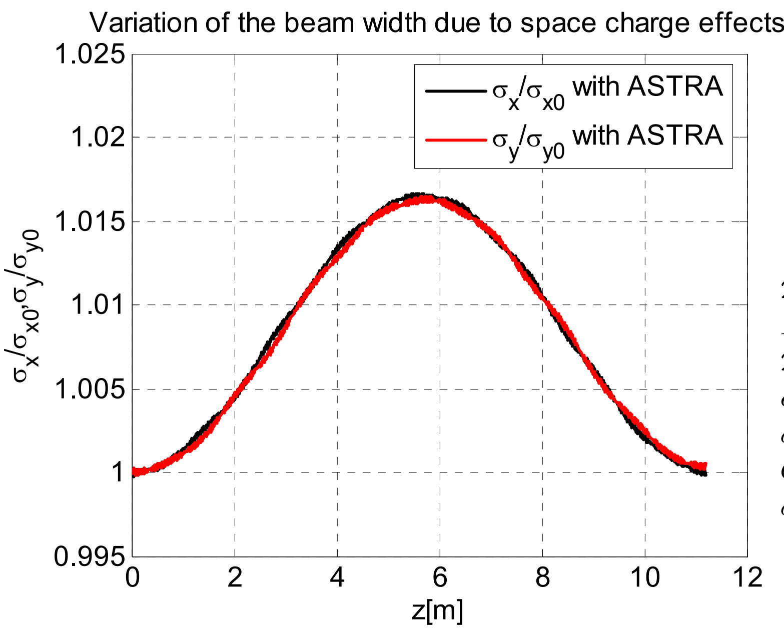
  - Image analysis



Systematical errors

- Error sources
  - Chromaticity
  - Space charge effects





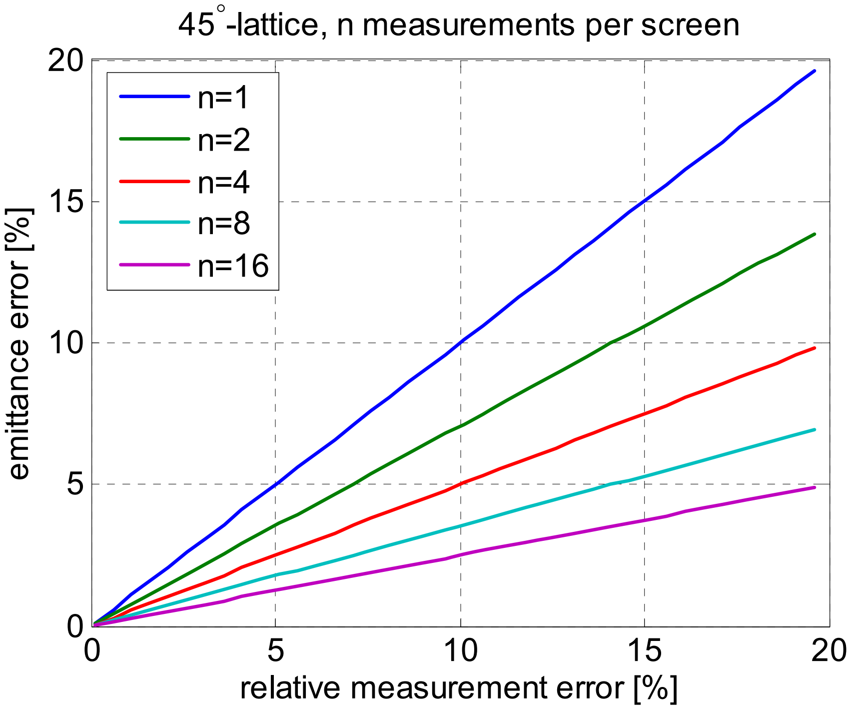
22.5°-lattice
$E = 500MeV$
2% energy spread
$\epsilon = 1\mu rad$
$\sigma = 50\mu m$
$Q = 1nC$
$\sigma_z = 100\mu m$

→ Simulation-based correction of the measured beam sizes

→ Emittance growth < 0.5%

Statistical errors

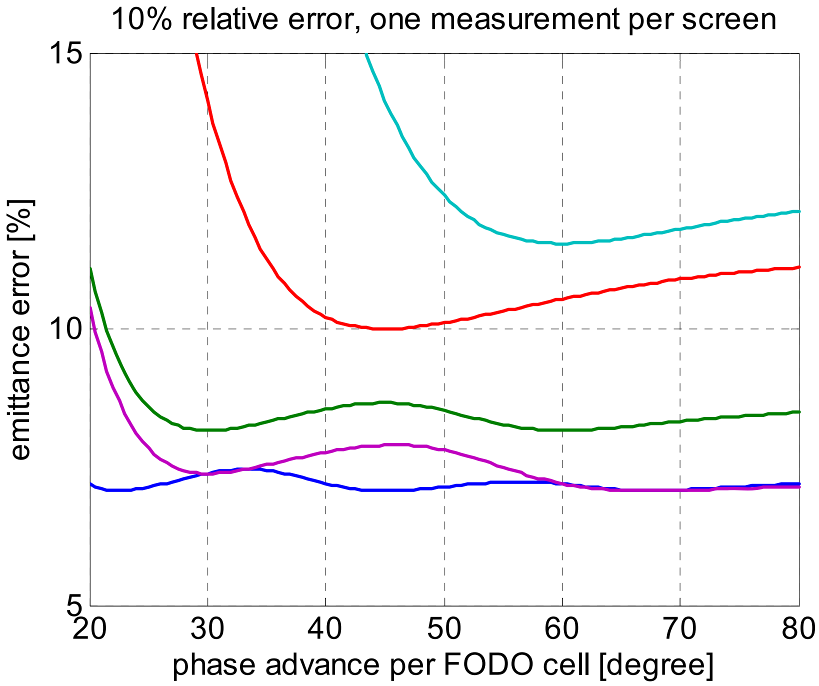
- **Error sources:**
  - Jitter of initial Twiss parameters
  - Image analysis
  - Jitter of beam energy
  - Limited resolution of the optical system
  - Fluctuation of sc-effects due to jitter of bunch shape and charge
  - Emittance Jitter (different analysis)



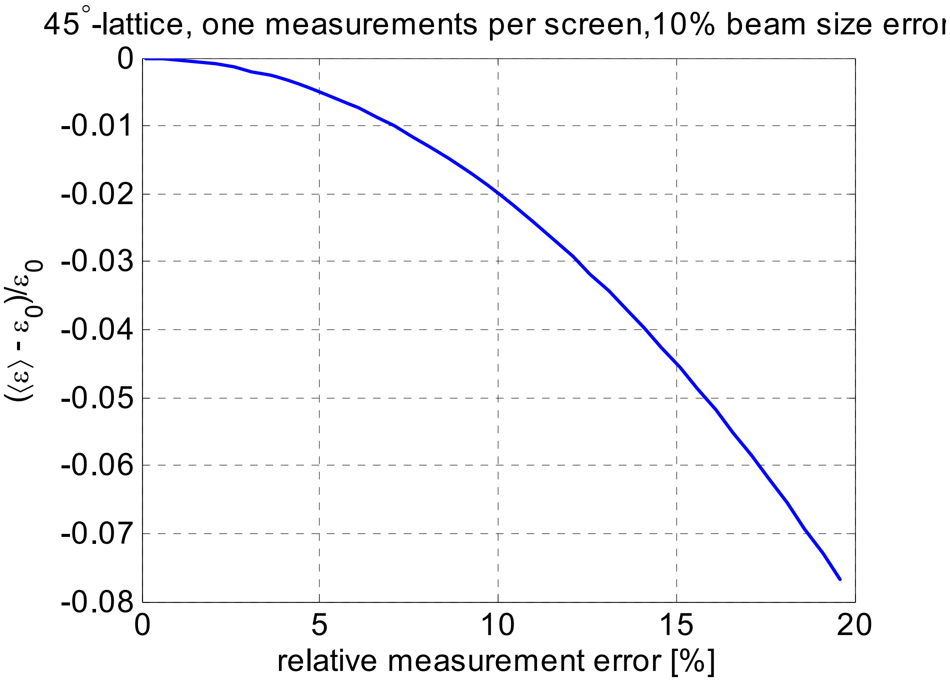
→ **No essential differences between the Lattices in case of statistical errors**

Statistical errors

Dependence on the phase advance per cell:



Deviation of the expectation value of the emittance:

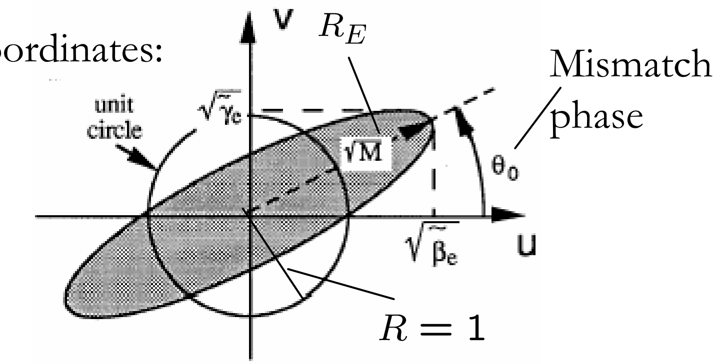


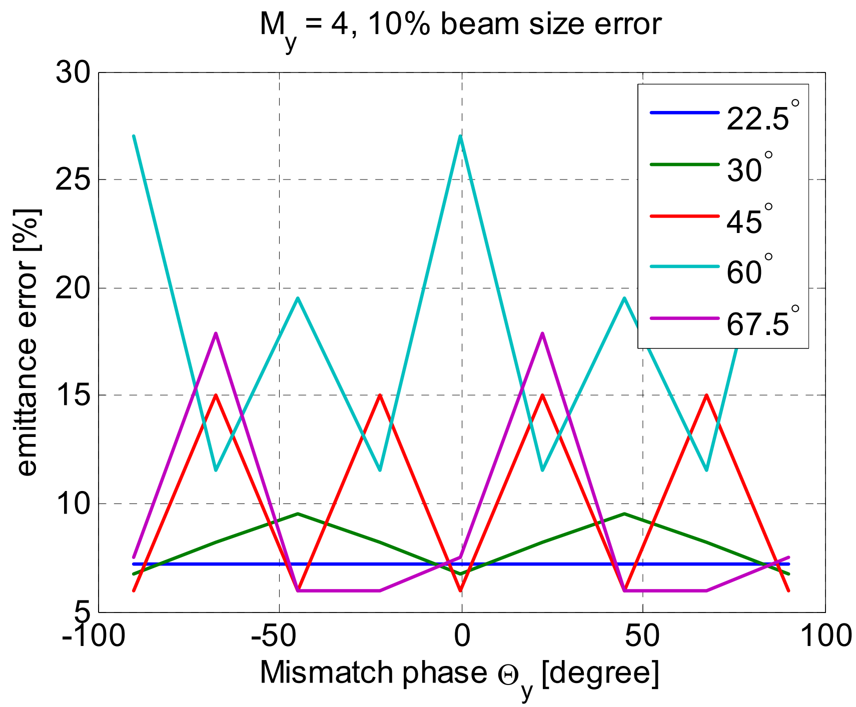
→ Averaging over beam sizes, not emittances

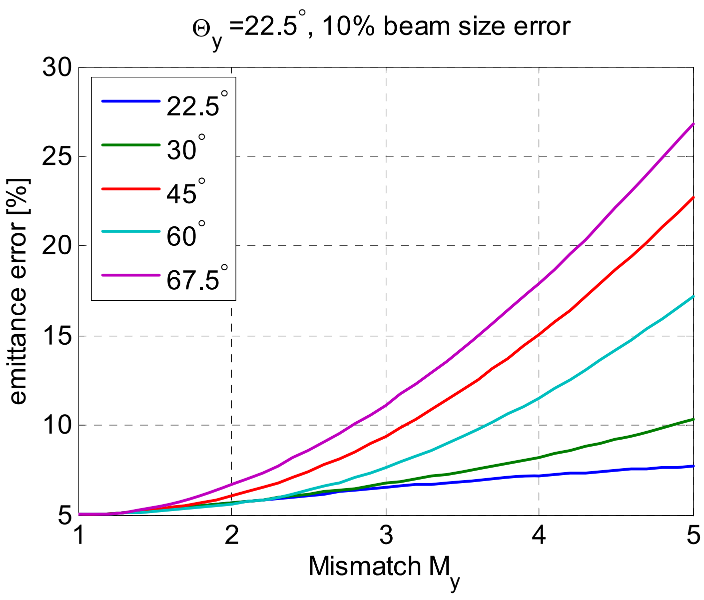
Statistical Errors: Measurements with a mismatched beam

- Mismatch parameter: $M = \left(\frac{R_E}{R}\right)^2$

Normal coordinates:







→ **22.5 °-lattice allows measurements with mismatched beams**

Coupling measurements

- 4-dimensional beam matrix: $\sigma = \begin{pmatrix} \sigma_x & \sigma_{xy} \\ \sigma_{xy} & \sigma_y \end{pmatrix}$

$\epsilon^{4d} = \epsilon_x \cdot \epsilon_y$ only for $\sigma_{xy} = 0$

→ In order to interpret the projected emittances we need in general to know the couplings

- **Coupling sources:** Transverse laser profile, Misalignments in gun section, role error of quadrupoles, residual dispersion, asymmetries in the cavities (Main coupler, HOM coupler), higher order magnetic fields, stray fields
- Measurement of $\sigma_{14} = \langle xy \rangle$ possible

Coupling measurements

- Dependence of σ_{14} on the initial couplings :

$$\sigma'_{14} = R \cdot \begin{pmatrix} \sigma_{13} \\ \sigma_{14} \\ \sigma_{23} \\ \sigma_{24} \end{pmatrix}$$

→ Same formalism as in case of projected emittance measurements
→ 180°-periodicity of σ_{14}
→ At least 5 measurements to allow a least square fit

→ 4-Screen-method is not the best choice for coupling measurements

Overview: Advantages and disadvantages of the 22.5°-lattice compared to the standard 45°-lattice

Advantages:

- More flexibility (mismatched beams, phase advance per cell)
- Smaller systematical errors (OTR-calibration errors, quadrupole gradient errors)
- Coupling measurements with least square fit method is possible
- 4-screen-method for fast measurements still available
- Availability (in case a CCD camera fails, 4-screen-method)

Disadvantages:

- More quadrupoles are needed
- Section is slightly longer
- Less space in drift sections
- The measurements take more time

Conclusions and Outlook

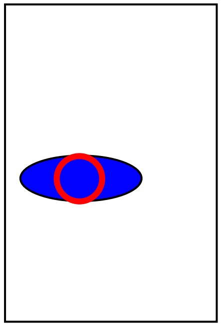
- A 22.5°-lattice seems to be the best solution from the considerations made so far for the first diagnostic section, a 45°-lattice for the one at 2 GeV
- To be considered in detail: Off-axis-measurements, slice emittance measurements, phase space tomography

Measurements with kickers

Bunches can optionally be kicked onto off-axis OTR-screens.

- Advantage: Single bunches can be picked out of the bunch train for parasitic emittance measurements
- With one kicker up to 3 OTR-screens can be reached .(bild)
- Emittance measurement: kick in x-direction, measurement in y-direction and vice versa;
  - Main additional error sources:
    - Quadrupole field errors
    - Variations of the kicks (~1%)

x



- Online coupling measurement problematic
- The beam width in kick direction depends on the 6 free parameters of

$$\sigma_0 = \begin{pmatrix} \langle x^2 \rangle & \langle x x' \rangle & \langle x\delta \rangle \\ \langle x' x \rangle & \langle x'^2 \rangle & \langle x'\delta \rangle \\ \langle \delta x \rangle & \langle \delta x' \rangle & \langle \delta^2 \rangle \end{pmatrix}$$

$\rightarrow$ online dispersion measurement possible

The Formalism of emittance measurements

- Residual vector $r = \Sigma - R \cdot \hat{o}$ provides information on the quality of the measurements
- The error of the solution $\hat{o}$ is determined by the covariavce matrix

$$V_{\hat{o}} = (R^t V_{\Sigma}^{-1} R)^{-1}$$

The Formalism of emittance measurements

- From $\sigma(s) = M\sigma(s_0)M^T$ one obtains the relation

$$\sigma_{1,1}(s)^2 = \begin{pmatrix} M_{1,1}^2 & 2M_{1,1}M_{1,2} & M_{1,2}^2 \end{pmatrix} \begin{pmatrix} \sigma_{1,1}(s_0)^2 \\ \sigma_{1,2}(s_0)^2 \\ \sigma_{2,2}(s_0)^2 \end{pmatrix}$$

- For n locations these equations can be combined to one matrix equation

$$\begin{pmatrix} (\sigma_{1,1}^{(1)})^2 \\ (\sigma_{1,1}^{(2)})^2 \\ \vdots \\ (\sigma_{1,1}^{(n)})^2 \end{pmatrix} = \begin{pmatrix} (M_{1,1}^{(1)})^2 & 2M_{1,1}^{(1)}M_{1,2}^{(1)} & (M_{1,2}^{(1)})^2 \\ (M_{1,1}^{(2)})^2 & 2M_{1,1}^{(2)}M_{1,2}^{(2)} & (M_{1,2}^{(2)})^2 \\ \vdots & & \\ (M_{1,1}^{(n)})^2 & 2M_{1,1}^{(n)}M_{1,2}^{(n)} & (M_{1,2}^{(n)})^2 \end{pmatrix} \begin{pmatrix} \sigma_{1,1}(s_0)^2 \\ \sigma_{1,2}(s_0)^2 \\ \sigma_{2,2}(s_0)^2 \end{pmatrix}$$

or

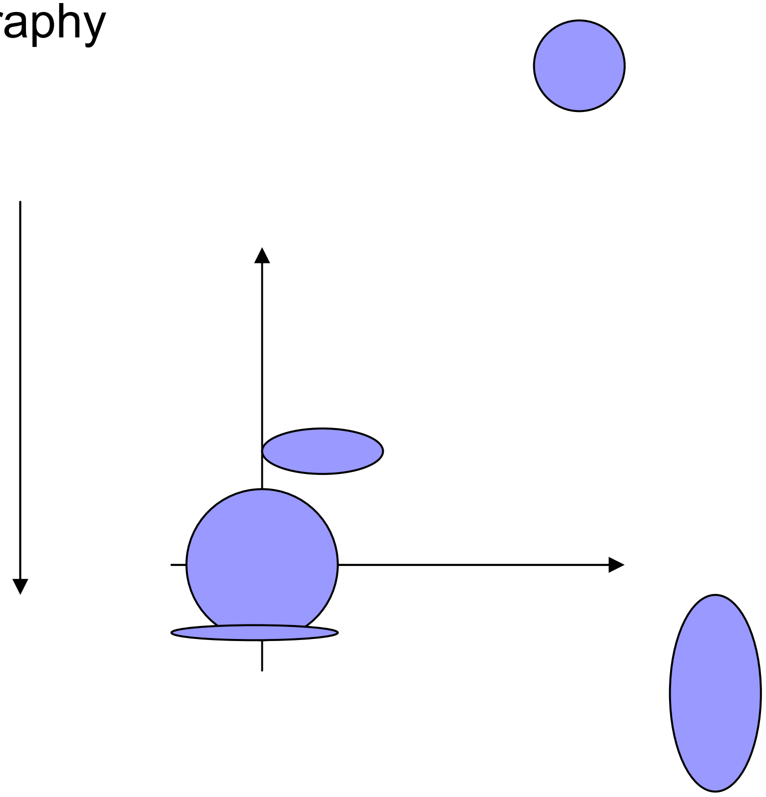
$$\Sigma = R \cdot o$$

- Determine solution $\hat{o}$ by least square fit method and calculate

$$\epsilon = \sqrt{\hat{\sigma}_{1,1}^2 \cdot \hat{\sigma}_{2,2}^2 - \hat{\sigma}_{1,2}^2}$$

Conclusions and Outlook

- Proposals for the diagnostic sections
- Tomography



Introduction

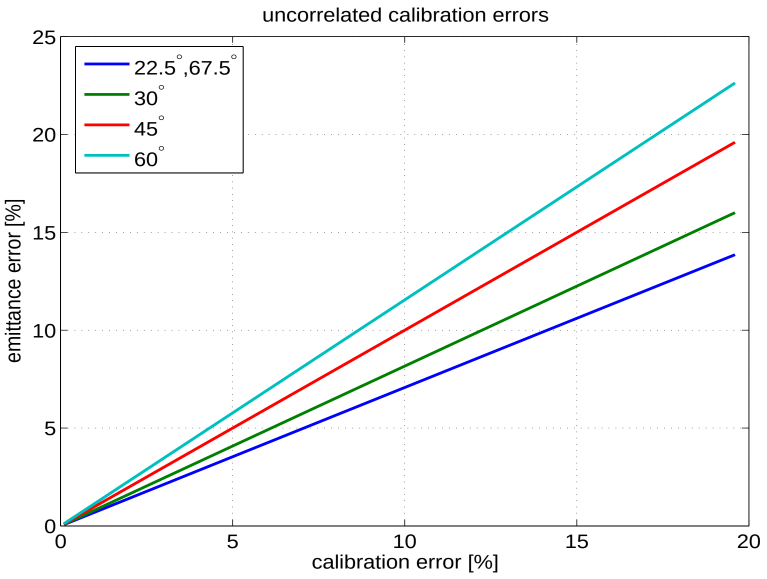
- Motivation …
- Objectives
  - Measurements should be online
  - Measurement of the projected emittances with an accuracy below 5%
  - Information about transverse couplings / 4-dimensional emittance
  - Emittance due to dispersion
  - Slice emittance measurements
  - Emittance variation over one bunch train
- Methods: Multi-monitor vs. quadrupole scan

Systematical errors

- Same OTR-calibration error / Systematical relative error in image analysis at all stations:

$$\frac{\sigma_\epsilon}{\epsilon} = 2 \cdot \frac{\sigma_{x_{rms}}}{x_{rms}}$$

- Statistically independent calibration errors / role angles of the cameras



Error analysis

- $\Sigma = R \cdot o$ $\rightarrow$ Two types of errors : $\Delta R,\ \Delta o$

$\rightarrow$ both types are equivalent in some sense

- Error sources:

Systematical errors	Statistical errors
Deviation of the beam energy	Jitter of beam energy/ initial Twiss parameters
Calibration of the OTR-monitors Role angles of the cameras	Limited resolution of the optical system
Image analysis	Image analysis (noise,rms-size)
Calibration of the quadrupole gradients	Fluctuation of sc-effects due to jitter of bunch shape and charge
Space charge effects, chromaticity	

In addition: Drifts, emittance jitter, initial mismatch

Arrangements

- Locations for kickers/ OTRs per kicker
- Traqnsverse deflecting cavities and kickers

$$\langle\epsilon\rangle \approx \epsilon(\langle o\rangle) + \frac{1}{2}\left\langle [\sum_{j=1}^{3} \Delta o_{j,1}\frac{\partial}{\partial o_{j,1}}]^2 \epsilon(o)|_{o=\langle o\rangle}\right\rangle$$

Emittance and Dispersion Measurements